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# DIOPHANTINE ANALYSIS.

101. Proposed by HARRY S. VANDIVER, Bala, Pa.

Prove that it is impossible to find integral values for  $x$ ,  $y$ , and  $z$  such that the relation  $x^2y + xz^2 = y^2z$  is satisfied.

II. Solution by W. F. KING, Ottawa, Canada.

We may assume that  $x$ ,  $y$ , and  $z$  have no factor common to them all. For if they have a common factor  $n$ , each term of the equation  $x^2y + xz^2 = y^2z$  may be divided by  $n^3$ , and there will then be left an equation of the same form, in which  $x$ ,  $y$ , and  $z$  have no common factor.

Let the greatest common measure of  $x$  and  $y$  be  $a$ ; of  $y$  and  $z$ ,  $b$ ; and of  $z$  and  $x$ ,  $c$ . Then we shall have

$$\begin{aligned}x &= cax_1, \\y &= aby_1, \\z &= bcz_1.\end{aligned}$$

Now observe that since  $b$  is the greatest common measure of  $y$  and  $z$ ,  $ay_1$  is prime to  $cz_1$ ; similarly is  $bz_1$  to  $ax_1$ , and  $cx_1$  to  $by_1$ .

Hence  $a$ ,  $b$ , and  $c$  are all prime to one another; so are  $x_1$ ,  $y_1$ , and  $z_1$  to one another. Also  $a$  is prime to  $z_1$ ,  $b$  to  $x_1$ , and  $c$  to  $y_1$ . Substituting in the equation the above values of  $x$ ,  $y$ , and  $z$ , and dividing by  $abc$ , we have

$$a^2c x_1^2 y_1 + c^2 b z_1^2 x_1 = b^2 a y_1^2 z_1.$$

$$\text{Divide by } x_1; \text{ then } a^2c x_1 y_1 + c^2 b z_1^2 = \frac{b^2 a y_1^2 z_1}{x_1}.$$

The left hand side is integral, therefore  $b^2 a y_1^2 z_1$  is divisible by  $x_1$ . But as above shown,  $x_1$  is prime to  $b$ ,  $y_1$ , and  $z_1$ . Hence  $a/x_1$  must be an integer.

Again, divide the above equation by  $a$ . Then

$$acx_1^2 y_1 + \frac{c^2 b z_1^2 x_1}{a} = b^2 y_1^2 z_1.$$

Hence, as before,  $\frac{c^2 b z_1^2 x_1}{a}$  is an integer. But  $a$  is prime to  $b$ ,  $c$ , and  $z_1$ .

Therefore,  $x_1/a$  is an integer. Now since  $a/x_1$  and  $x_1/a$  are both integers,  $x_1 = a$ . Similarly,  $y_1 = b$ ,  $z_1 = c$ . Therefore the equation

$$a^2c x_1^2 y_1 + c^2 b z_1^2 x_1 = b^2 a y_1^2 z_1$$

becomes  $a^4bc + abc^4 = ab^4c$ , or dividing by  $abc$ ,  $a^3 + c^3 = b^3$ , a known impossible form.